Hyperspectral Images Mixed Noise Removal Via Group-Tube Transform Induced Sparsity and Low-Rankness

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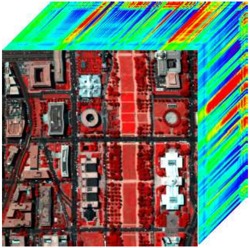
Outline

1. Background and Motivation
2. The Proposed Model and Algorithm
3. Experimental results
4. Conclusion

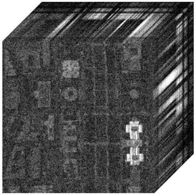
Hyperspectral Images (HSIs)

Hyperspectral Images (HSIs) contain wealthy spatial-spectral knowledge and have been widely used in many applications, such as material identification, mineral detection, and forest inspection.

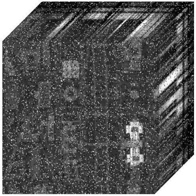


Degradation of HSIs

Due to the limitations of imaging devices and environment, HSIs in real applications always suffer from various noises, such as Gaussian noise, sparse noise, and stripes.



Gaussian noise



Sparse noise

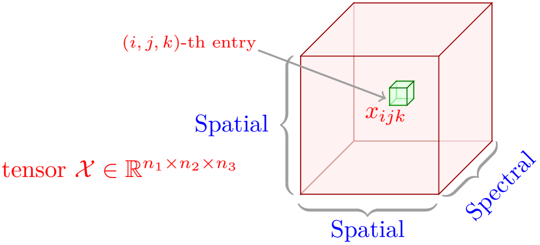
Mainstream priors for HSIs

HSIs have rich spatial-spectral correlations that can be leveraged for effective HSIs denoising:

1. piecewise smoothness;
2. nonlocal self-similarity;
3. low-rankness;
4. $\cdots$

Tensor singular values decomposition

HSI can be naturally represented by a third-order tensor $\mathcal{X}$ with two spatial dimensions and one spectral dimension.



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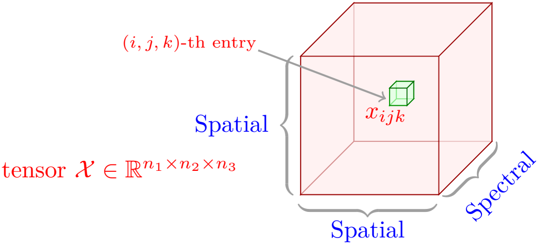


Tensor singular values decomposition (t-SVD) can capture the low-rankness of the third-order tensor, which has obtained the promising results for HSIs denoising.

Misha E. Kilmer and Carla D. Martin, Factorization strategies for third-order tensors, Linear Algebra and its Applications, vol. 435, pp. 641-658, 2011.

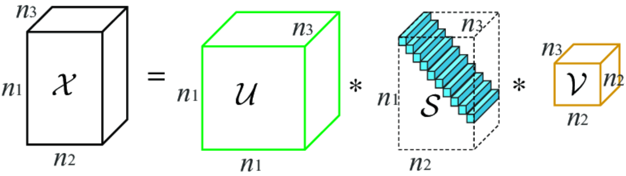
Tensor singular values decomposition

t-SVD and tubal-rank

The t-SVD of $\mathcal{X}$ is

$$\mathcal{X} = \mathcal{U} * \mathcal{S} * \mathcal{V}^H,$$

where $\mathcal{U}$ and $\mathcal{V}$ are orthogonal tensors, $\mathcal{S}$ is the f-diagonal tensor, and $\mathcal{V}^H$ denotes the conjugate transpose of $\mathcal{V}$. Herein, the tubal rank of $\mathcal{X}$ is defined as the number of non-zero tubes of $\mathcal{S}$.



Tensor nuclear norm

Tensor nuclear norm (TNN)

The TNN of $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ is denoted by $\|\mathcal{X}\|_{TNN}$, which is defined as

$$\|\mathcal{X}\|_{TNN} = \sum_{k=1}^{n_3} \|\mathbf{Z}_k\|_*,$$

where $\mathbf{Z}_k \in \mathbb{C}^{n_1 \times n_2}$ is the kth frontal slice of $\mathcal{Z} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$, and $\mathcal{Z} = \mathcal{X} \times_3 \mathbf{F}_{n_3}$ is the transformed tensor of $\mathcal{X}$ under the discrete Fourier transform (DFT) $\mathbf{F}_{n_3} \in \mathbb{C}^{n_3 \times n_3}$, and $\times_3$ denotes the matrix-tensor product.

C. Lu, J. Feng, Y. Chen, W. Liu, Z. Lin and S. Yan, Tensor Robust Principal Component Analysis with a New Tensor Nuclear Norm, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 42, no. 4, pp. 925-938, 2020.

Group-tube transform based TNN

Group-tube transform based TNN (GTNN)

Given a target tensor $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, its GTNN is denoted by $\|\mathcal{X}\|_{\text{GTNN}}$. Formally, we have

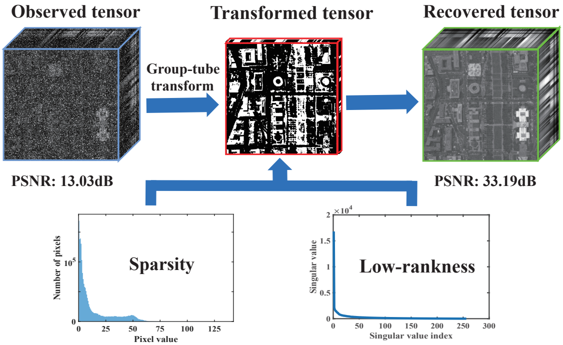
$$\|\mathcal{X}\|_{\mathrm{GTNN}} \triangleq \sum_{k=1}^{\tilde{n}_3} \|\mathbf{Z}_k\|_*, \quad k = 1, \ldots, \tilde{n}_3$$

where $\mathbf{Z}_k = \sum_{j=1}^{w} \mathbf{W}_{k,j} \circledast (\mathcal{X} \times_3 \mathbf{D})_k$ is the kth frontal slice of $\mathcal{Z} \in \mathbb{R}^{n_1 \times n_2 \times \tilde{n}_3}$, $\mathcal{Z}$ is the transformed tensor, $(\mathcal{X} \times_3 \mathbf{D})_k$ is the k th frontal slice of $\mathcal{X} \times_3 \mathbf{D}$, $\mathbf{W}_{k,j}$ is the 2D filters, and $\circledast$ denotes the convolution.

B.-Z. Li, X.-L. Zhao, X. Zhang, T.-Y. Ji, X. Chen, Michael K. Ng, A Learnable Group-Tube Transform Induced Tensor Nuclear Norm and Its Application for Tensor Completion, SIAM Journal on Imaging Sciences, vol. 16, pp. 1370-1397, 2023.

Transformed Sparsity and Low-rankness

The transform-based TNN methods only exploit the transformed low-rankness.



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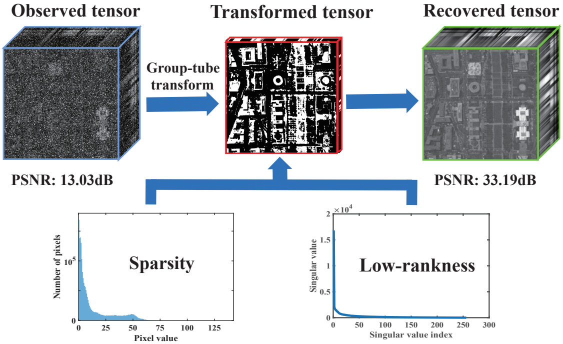


The sparsity of the transformed tensor is neglected.

GTCSLR

Given a noisy tensor $\mathcal{Y} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$, we propose group-tube transform induced collaborative sparsity and low-rankness (GTCSLR) for HSI mixed noise removal:

$$\begin{aligned}
\min_{\substack{\mathcal{X},\mathcal{Z},\mathcal{E},\mathcal{S}\\ \mathcal{M},\mathbf{D},\mathbf{W}}} & \sum_{k=1}^{\tilde{n}_3} \left(\|\mathbf{Z}_k\|_* + \lambda_1 \|\mathbf{E}_k\|_1 \right) + \lambda_2 \|\mathcal{S}\|_1 \\
\text{s.t. } & \|\mathcal{Y} - \mathcal{X} - \mathcal{S}\|_F^2 \leq \epsilon, \mathcal{X} = \mathcal{M} \times_3 \mathbf{D}^\top, \\
& \mathbf{Z}_k = \mathbf{W}_k \circledast \mathbf{M}_k, \mathbf{E}_k = \mathbf{W}_k \circledast \mathbf{M}_k, \\
& \text{for } k = 1, 2, \cdots, \tilde{n}_3,
\end{aligned}$$

where $\mathcal{X}$ is the clean HSI, $\mathcal{S}$ is the sparse noise, $\mathcal{M}$ is the spectral transformed tensor, $\mathbf{M}_k$ is the k th frontal slice of $\mathcal{M}$, $\mathcal{Z}$ is the low-rank tensor, $\mathcal{E}$ is the sparse tensor, and $\mathbf{Z}_k$ and $\mathbf{E}_k$ is k-th frontal slice of $\mathcal{Z}$ and $\mathcal{E}$, respectively.

PAM algorithm for GTCSLR

We update each variable alternatively under proximal alternating minimization (PAM) algorithm framework.

- $\mathbf{Z}_k$ subproblem:

$$\mathbf{Z}_k^{t+1} = \mathcal{T}_{\frac{1}{\gamma+\rho}}\Big(\frac{\gamma \mathbf{W}_k^t \circledast \mathbf{M}_k^t + \rho \mathbf{Z}_k^t}{\gamma+\rho}\Big),\ k = 1, \cdots, \tilde{n}_3,$$

- $\mathbf{E}_k$ subproblem:

$$\mathbf{E}_k^{t+1} = \mathscr{S}_{\frac{\lambda_1}{\mu+\rho}}\Big(\frac{\mu \mathbf{W}_k^t \circledast \mathbf{M}_k^t + \rho \mathbf{E}_k^t}{\gamma+\rho}\Big),\ k = 1, \cdots, \tilde{n}_3.$$

- $\mathcal{S}$ subproblem:

$$\mathcal{S}^{t+1} = \mathscr{S}_{\frac{\lambda_2}{\mu+\rho}}\Big(\frac{\alpha(\mathcal{Y} - \mathcal{X}^{t+1}) + \rho \mathcal{S}^t}{\alpha+\rho}\Big).$$

PAM algorithm for GTCSLR

- $\mathcal{M}$ subproblem:

$$\mathcal{M}^{t+1} = \mathcal{F}^{-1}\Big(\frac{(\gamma+\mu)\mathcal{F}^{-1}(\mathbf{W}_k^t)\mathcal{F}(\mathbf{H}_k^t) + (\beta+\rho)\mathcal{F}(\mathbf{R}_k^t)}{(\gamma+\rho)\mathcal{F}^{-1}(\mathbf{W}_k^t)\mathcal{F}(\mathbf{W}_k^t) + (\beta+\rho)\mathbf{I}}\Big),$$

where $\mathbf{H}_k^t = \frac{\gamma\mathbf{Z}_k^{t+1}+\mu\mathbf{E}_k^{t+1}}{\gamma+\mu}$ and $\mathbf{R}_k^t = \frac{\beta(\mathcal{X}^{t+1}\times_3\mathbf{D}^t)_k+\rho\mathbf{M}_k^t}{\beta+\rho}$;

- $\mathbf{W}_k$ subproblem:

$$\mathbf{W}_{k,j}^{t+1} = \mathcal{F}^{-1}\Big(\frac{(\gamma+\mu)\mathcal{F}^{-1}(\mathbf{Q}_k^t)\mathcal{F}(\mathbf{M}_k^{t+1}) + (\beta+\rho)\mathcal{F}(\mathbf{R}_k^t)}{(\gamma+\rho)\mathcal{F}^{-1}(\mathbf{Q}_k^t)\mathcal{F}(\mathbf{Q}_k^t) + (\beta+\rho)\mathbf{I}}\Big),$$

where $\mathbf{Q}_k^t = \frac{\gamma\mathbf{Z}_k^{t+1}+\mu\mathbf{E}_k^{t+1}}{\gamma+\mu} - \sum_{i\neq j}\mathbf{W}_{k,j}\circledast\mathbf{M}_k^{t+1},\ j=1,\cdots,w;$

PAM algorithm for GTCSLR

- $\mathcal{X}$ subproblem:

$$\mathcal{X}^{t+1} = \frac{\alpha(\mathcal{Y} - \mathcal{S}^t) + \beta\mathcal{M}^t \times_3 \mathbf{D}^{t\top} + \rho\mathcal{X}^t}{\alpha + \beta + \gamma}$$

- $\mathbf{D}$ subproblem:

$$\mathbf{D}^{t+1} = \mathbf{V}\mathbf{U}^\top,$$

where $\mathbf{U}$ and $\mathbf{V}$ are left and right singular vectors of the following SVD:

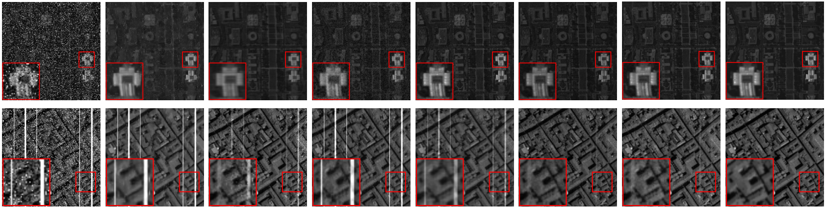
$$\beta\mathbf{X}_{(3)}^{t+1}(\mathbf{M}_{(3)})^\top + \rho\mathbf{D}^t = \mathbf{U}\Sigma\mathbf{V}^\top.$$

Compared Methods and Datasets

- Compared methods:
  1. BM4D [Maggioni et al. IEEE TIP 2013]
  2. LRMR [Zhang et al. IEEE TGRS 2014]
  3. LRTR [Fan et al. IEEE J-STARS 2017]
  4. 3DTNN [Zheng et al. IEEE TGRS 2020]
  5. GTNN [Li et al. SIIMS 2023]
- Dtasets:
  1. Washington DC Mall $(256 \times 256 \times 100)$
  2. Pavia City $(200 \times 200 \times 80)$
- Metrics: PSNR $\uparrow$, SSIM $\uparrow$, and SAM $\downarrow$.

Numerical results

Datasets	Washington DC Mall						Pavia City					
Cases	Case 1			Case2			Case 1			Case2		
Indicators	PSNR	SSIM	SAM	PSNR	SSIM	SAM	PSNR	SSIM	SAM	PSNR	SSIM	SAM
Noisy	13.752	0.2239	34.928	13.032	0.1854	37.219	14.218	0.2453	40.776	13.255	0.1882	42.528
BM4D	26.876	0.7378	11.432	24.289	0.6647	15.236	25.590	0.7153	13.226	23.245	0.6849	16.884
LRMR	33.639	0.9402	4.173	30.966	0.8704	4.664	32.133	0.8920	7.016	30.361	0.8288	11.570
LRTR	34.095	0.9533	2.640	31.789	0.8907	4.105	35.322	0.9635	4.380	27.113	0.8396	17.301
3DTNN	35.973	0.9617	3.361	32.956	0.9076	4.036	35.208	0.9554	6.084	28.285	0.8608	11.073
GTNN	36.422	0.9645	2.261	32.782	0.9092	4.255	38.059	0.9742	4.481	31.669	0.9190	7.602
GTCSLR	**37.068**	**0.9679**	**2.110**	**33.190**	**0.9139**	**4.005**	**38.411**	**0.9752**	**4.323**	**32.144**	**0.9226**	**6.846**



Observed BM4D LRMR LRTR 3DTNN GTNN GTCSLR Original

Conclusion

1. We propose the group-tube transform induced collaborative sparsity and low-rankness (GTCSLR) for HSIs mixed noise removal model, which is capable of simultaneously characterizing low-rankness and sparsity of the transformed tensor.
2. We develop an efficient PAM algorithm to solve the proposed non-convex model. Numerical experiments demonstrated the superiority of the proposed method compared with the state-of-the-art HSIs denoising methods.

Thanks for your attention!

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